

# High-energy physics with particles carrying non-zero orbital angular momentum

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**Abstract** Thanks to progress in optics in the past two decades, it is possible to create photons carrying well-defined non-zero orbital angular momentum (OAM). Boosting these photons into high-energy range preserving their OAM seems feasible. Intermediate energy electrons with OAM have also been produced recently. One can, therefore, view OAM as a new degree of freedom in high-energy collisions and ask what novel insights into particles' structure and interactions it can bring. Here we discuss generic features of scattering processes involving particles with OAM in the initial state. We show that they make it possible to perform a Fourier analysis of a plane wave cross section with respect to the azimuthal angles of the initial particles, and to probe the autocorrelation function of the amplitude, a quantity inaccessible in plane wave collisions.

**Keywords** Orbital angular momentum

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## 1 Introduction

Laser beams carrying non-zero orbital angular momentum (OAM) are well-known and routinely used in optics, [1]. The lightfield in such beams is described via non-plane wave solutions of the Maxwell equations, for example by Bessel or Laguerre-Gaussian beams. Each photon in this lightfield, which we call a *twisted photon*, carries a well-defined energy and longitudinal momentum directed along an arbitrarily chosen axis  $z$  as well as a definite OAM projection onto this axis quantized in units of  $\hbar$ . The wavefronts of such a lightfield are not planes but helices.

So far, experiments with twisted light were confined mostly to the optical energy range. However it was recently noted that one can use Compton backscattering of twisted optical photons off an ultra-relativistic electron beam to generate high-energy photons carrying

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non-zero OAM [2, 3]. Technology necessary for such conversion already exists. In addition, successful creation of twisted electrons has also been reported recently, [4]. Such electrons carried the energy as high as 300 keV and the orbital quantum number up to  $m \sim 100$ . One can imagine that future progress in this field will lead to creation of even more energetic twisted electrons and other particles, which can then be used in scattering experiments. Thus, OAM can be viewed as a new degree of freedom which one can exploit in preparing initial states of a high-energy process. It is therefore timely to ask how such a collision can be described and what new insights into the properties of particles and their interactions it can bring.

In this contribution we consider generic features of scattering processes involving twisted particles in the initial state. Specifically, we consider two cases — single-twisted scattering (collision of a twisted state with a plane wave) and double-twisted scattering (collision of two coaxial twisted states) — and derive the cross sections of these processes in terms of the corresponding plane wave cross sections. More details about the results presented here can be found in [5].

## 2 Describing twisted states

### 2.1 Scalar case

Here we briefly summarize the formalism of Bessel-beam twisted states introduced in [2] starting with the scalar field.

We first fix the  $z$  axis and solve the free wave equation in cylindric coordinates  $r, \varphi_r, z$ . A solution  $|\kappa, m\rangle$  with definite frequency  $\omega$ , longitudinal momentum  $k_z$ , modulus of the transverse momentum  $|\mathbf{k}| = \kappa$  and a definite  $z$ -projection of the orbital angular momentum  $m$  has the form

$$|\kappa, m\rangle = e^{-i\omega t + ik_z z} \cdot \psi_{\kappa m}(\mathbf{r}), \quad \psi_{\kappa m}(\mathbf{r}) = \frac{e^{im\varphi_r}}{\sqrt{2\pi}} \sqrt{\kappa} J_m(\kappa r), \quad (1)$$

where  $J_m(x)$  is the Bessel function. The transverse spatial distribution is normalized according to

$$\int d^2\mathbf{r} \psi_{\kappa' m'}^*(\mathbf{r}) \psi_{\kappa m}(\mathbf{r}) = \delta_{mm'} \sqrt{\kappa \kappa'} \int r dr J_m(\kappa r) J_m(\kappa' r) = \delta_{mm'} \delta(\kappa - \kappa'). \quad (2)$$

A twisted state can be represented as a superposition of plane waves:

$$|\kappa, m\rangle = e^{-i\omega t + ik_z z} \int \frac{d^2\mathbf{k}}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) e^{i\mathbf{k}\mathbf{r}}, \quad (3)$$

where

$$a_{\kappa m}(\mathbf{k}) = (-i)^m e^{im\varphi_k} \sqrt{2\pi} \frac{\delta(|\mathbf{k}| - \kappa)}{\sqrt{\kappa}}. \quad (4)$$

This expansion can be inverted:

$$e^{i\mathbf{k}\mathbf{r}} = \sqrt{\frac{2\pi}{\kappa}} \sum_{m=-\infty}^{+\infty} i^m e^{-im\varphi_k} |\kappa, m\rangle, \quad \kappa = |\mathbf{k}|. \quad (5)$$

From (3) one can simply state that the twisted state is a steady interference pattern arising from all plane waves with fixed  $|\mathbf{k}|$  and arriving from different directions.

More details about properties of twisted states, their normalization and phase space density can be found in [3,5]. Here we just note that although the wave oscillation amplitude decreases at large radii, the pure Bessel twisted state of finite amplitude is still not localized and not normalizable in the transverse plane. Therefore, the intermediate calculations with these Bessel-beam states must be carried out inside a large but finite cylindric volume of radius  $R$ .

## 2.2 Twisted photons

Description of photons carrying non-zero OAM is subtler than for scalar particles due to their polarization degree of freedom. A plane wave photon with helicity  $\lambda = \pm 1$  is described, in addition to the fixed four-momentum  $k^\mu$ , by an appropriately defined polarization vector  $e_\lambda^\mu(k)$ , with the properties  $e_{\lambda\mu} k^\mu = 0$  and  $e_{\lambda\mu}^* e_{\lambda'\mu}^\mu = -\delta_{\lambda\lambda'}$ . In the plane wave case, the polarization vector appears as an overall factor in front of the space-time wave function: the components of the polarization vector, which can be selected to be only transverse, remain constant across the transverse plane orthogonal to the Poynting vector. The same is valid for the Stokes parameters for a general elliptic polarization state.

In the twisted case both the polarization vector of a pure helicity state and the Stokes parameters of an elliptically polarized state acquire non-trivial spatial dependence. Even worse, the polarization vectors taken at different points cannot lie in the same plane because the directions of the Poynting vector calculated at distinct spatial points are different.

One can represent a pure helicity twisted photon state in the coordinate space similarly to (3):

$$A_{\lambda\kappa m}^\mu(x) = \sqrt{4\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} e_\lambda^\mu(k) a_{\kappa m}(\mathbf{k}) e^{-ik_\mu r^\mu}, \quad (6)$$

Even with the four-potential (6) depending non-trivially on the coordinates, the gauge invariance in its usual definition as an invariance under  $A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu f(x)$  still holds. Note however that the definition (6) should be accompanied with a prescription of how vectors  $e_\lambda^\mu(k)$  for different  $k$  are related to each other. Recall that for the plane waves with  $\mathbf{k} = (0, 0, k)$ , the polarization vector is defined up to an overall phase:

$$\mathbf{e}_\lambda = -\frac{1}{\sqrt{2}}(\lambda, i, 0) \cdot e^{i\alpha}, \quad (7)$$

but the (arbitrary)  $\alpha$  disappears in the matrix elements squared. This phase shift is equivalent to the shift of the zero moment of time.

This can be repeated for each plane wave inside a twisted state. If the three-momentum  $\mathbf{k} = \omega(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ , we can introduce, following [6], the unit vectors  $\mathbf{e}_\theta, \mathbf{e}_\phi$  and construct circular polarizations as

$$\mathbf{e}_\lambda(\mathbf{k}) = -\frac{1}{\sqrt{2}}(\lambda\mathbf{e}_\theta + i\mathbf{e}_\phi) \cdot e^{i\alpha(\phi)}, \quad (8)$$

where  $\alpha(\phi)$  is, in principle, an arbitrary periodic function, which, however, does not affect the physical observables. We choose  $\alpha(\phi) = \lambda\phi$  which yields the correct paraxial limit: when  $\kappa \rightarrow 0$ , the polarization vector (8) turns into (7).

Let us also note in passing that the polarization states of twisted photons are much richer than for the plane waves. The Stokes parameters describing the polarization locally can change from point to point, so that one must deal with *polarization field* rather than

polarization parameters. The polarization field is even allowed to have singularities at the transverse origin, where the intensity of the lightfield is zero for any  $m \neq 0$ . Such singular lightfields are also well-known in optics, see e.g. [7].

At this point it is necessary to address the issue of spin-OAM separation, which (in the non-abelian case of QCD) is a hot topic in the high-energy physics community, especially in the context of the spin proton puzzle [8]. Here we talk about photons with a definite value of OAM and a definite helicity  $\lambda$ . However this is not the spin/OAM separation that one usually has in mind due to two reasons: (1) this is a separation of spin and OAM degrees of freedom not at the level of operators, but at the level of their average values over certain states, and (2) the average values of only  $z$  components of these operators are involved. At this level, the possibility to separate these two degrees of freedom is not unexpected, see e.g. a detailed discussion in [9]. Let us also mention that for the paraxial twisted light beams the separation of OAM and helicity is also easily derived, see [10]. For non-paraxial beams this issue is more tricky; in this case the evolution of OAM and helicity in the course of beam propagation can be cast into the form of an effective spin-orbital interaction, [6].

### 3 Single-twisted scattering

Let us now consider a generic collision of a twisted particle with a plane wave:

$$|\kappa, m\rangle + |PW(\mathbf{p})\rangle \rightarrow X. \quad (9)$$

The final system  $X$  is assumed to be describable by plane waves. The passage from the plane wave to the twisted state is given by (3) and is applied at the level of scattering matrix:

$$S_{tw} = \int \frac{d^2\mathbf{k}}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) S_{PW}(\mathbf{k}, \mathbf{p}). \quad (10)$$

Its square is

$$\begin{aligned} |S_{tw}|^2 &= \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d^2\mathbf{k}'}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) a_{\kappa m}^*(\mathbf{k}') S_{PW}(\mathbf{k}, \mathbf{p}) S_{PW}^*(\mathbf{k}', \mathbf{p}) \\ &\propto \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d^2\mathbf{k}'}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) a_{\kappa m}^*(\mathbf{k}') \delta^{(2)}(\mathbf{k} + \mathbf{p} - \mathbf{p}_X) \delta^{(2)}(\mathbf{k}' + \mathbf{p} - \mathbf{p}_X) \mathcal{M}(\mathbf{k}, \mathbf{p}) \mathcal{M}^*(\mathbf{k}', \mathbf{p}) \\ &= \int \frac{d^2\mathbf{k}}{(2\pi)^4} a_{\kappa m}(\mathbf{k}) a_{\kappa m}^*(\mathbf{k}) \delta^{(2)}(\mathbf{k} + \mathbf{p} - \mathbf{p}_X) |\mathcal{M}(\mathbf{k}, \mathbf{p})|^2. \end{aligned}$$

Here  $\mathbf{p}_X$  is the transverse momentum of the final system  $X$ . The last line here contains the expression that enters the plane wave cross section of the same process. Skipping details which can be found in [5], we give the final result which links the single-twisted cross section to the plane wave cross section:

$$d\sigma_{tw} = \int \frac{d\phi_k}{2\pi} d\sigma_{PW}(\mathbf{k}) \cdot \frac{j_{PW}(\mathbf{k})}{j_{tw}}. \quad (11)$$

Here  $j_{PW}$  and  $j_{tw}$  are the plane-wave and the twisted flux functions. Discussions on subtle aspects of the definitions of the (averaged) cross section and the flux functions for the twisted scattering can be found in [3, 5]. Here we just note that the ratio of the fluxes in (11) is very close to unity for  $\kappa$  values achievable with today's technology.

The expression (11) is remarkable in several aspects. First, it is an unusual quantity in the sense that it involves averaging over the initial particle's azimuthal angle at fixed final momenta. Second, the single-twisted cross section is  $m$ -independent, which proves that twisted particles scatter as easily as plane waves. Third, the cross section (11) stays finite in the small  $\kappa$  limit. Fourth,  $d\sigma_{tw}$  is represented as an incoherent sum of  $d\sigma_{PW}(\mathbf{k})$  for different angles  $\phi_k$ , although the initial twisted state itself is a coherent superposition. The initial coherence is lost not during the interaction itself, but as a result of the usual condition that final states with distinct momenta do not interfere in the incoherent detectors we have.

Let us now consider the same process (9) but assume that the twisted particle is in a superposition of states with different  $m$ , for example,  $a|\kappa, m\rangle + a'|\kappa, m'\rangle$ , with  $|a|^2 + |a'|^2 = 1$ . The calculation can be repeated yielding

$$d\sigma = d\sigma_{tw} + 2|aa'|d\sigma_{tw}^{\Delta m}. \quad (12)$$

where  $d\sigma_{tw}$  is given by (11) and the new term is

$$d\sigma_{tw}^{\Delta m} = \int \frac{d\phi_k}{2\pi} \cos(\Delta m \phi_k + \alpha) d\sigma_{PW}(\mathbf{k}) \cdot \frac{j_{PW}(\mathbf{k})}{j_{tw}}, \quad (13)$$

with  $\Delta m = m - m'$  and  $\alpha$  being the relative phase between  $a$  and  $a'$ .

Looking at (11) and (13) can observe that with a well controlled  $m$  distribution of the initial twisted state one can perform the Fourier-analyzer of the plane wave cross section with respect to the initial azimuthal angle  $\phi_k$ .

What can be a potential application of this new tool? Imagine a typical elastic scattering of a probe particle on a (polarized) target of an unknown structure. Let the initial and final transverse momenta of the probe particle be  $\mathbf{k}$  and  $\mathbf{k}'$ , respectively, with  $\mathbf{q} = \mathbf{k} - \mathbf{k}'$  being the momentum transfer. If we are interested in the elastic formfactor, we study the  $\mathbf{q}^2$ -dependence of the cross section by measuring scattering at different final polar angles of the vector  $\mathbf{k}'$ . We can be also interested in azimuthal dependence of the  $\mathbf{k}'$ -distribution with respect, for example, to the transverse polarization direction  $\mathbf{n}$  of the target, which would allow us to measure the  $\mathbf{qn}$  dependence.

With the new tool, one can, in principle, access the same  $\mathbf{q}^2$ -dependent and  $\mathbf{qn}$ -dependent terms with *fixed* final momentum  $\mathbf{k}'$ . Of course, the resolving power depends on how exactly the initial twisted state can be prepared, but being a completely different experimental set-up, it might become an interesting complementary tool.

#### 4 Double-twisted cross section

Let us now consider a collision of two twisted particles:

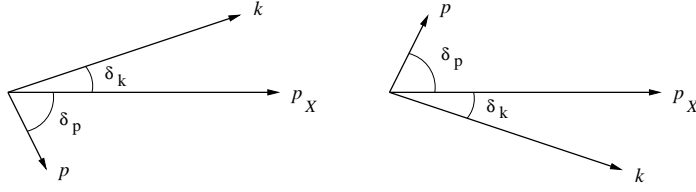
$$|\kappa, m\rangle + |\eta, n\rangle \rightarrow X. \quad (14)$$

Now, the scattering matrix is

$$S_{2tw} = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d^2\mathbf{p}}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) a_{\eta n}(\mathbf{p}) S_{PW}(\mathbf{k}, \mathbf{p}), \quad (15)$$

and its square contains

$$\begin{aligned} & \int \frac{d^2\mathbf{k} d^2\mathbf{p} d^2\mathbf{k}' d^2\mathbf{p}'}{(2\pi)^8} a_{\kappa m}(\mathbf{k}) a_{\eta n}(\mathbf{p}) a_{\kappa m}^*(\mathbf{k}') a_{\eta n}^*(\mathbf{p}') \\ & \times \delta^{(2)}(\mathbf{k} + \mathbf{p} - \mathbf{p}_X) \delta^{(2)}(\mathbf{k}' + \mathbf{p}' - \mathbf{p}_X) \mathcal{M}(\mathbf{k}, \mathbf{p}) \mathcal{M}^*(\mathbf{k}', \mathbf{p}'). \end{aligned}$$



**Fig. 1** Two kinematical configurations of the transverse momenta  $\mathbf{k}$  and  $\mathbf{p}$  of fixed absolute values that sum up to the vector  $\mathbf{p}_X$ .

Trying to satisfy all the kinematical restrictions which enter this expression at fixed final transverse momentum  $\mathbf{p}_X$ , we end up with exactly two kinematical configurations shown in Fig. 1. These two configurations are at work both for  $\mathcal{M}(\mathbf{k}, \mathbf{p})$  and the conjugate of  $\mathcal{M}(\mathbf{k}', \mathbf{p}')$ , which means that two relations between  $\mathbf{k}, \mathbf{p}$  and  $\mathbf{k}', \mathbf{p}'$  are possible:

$$\begin{aligned} \text{direct: } \mathbf{k}' &= \mathbf{k}, \mathbf{p}' = \mathbf{p}, \\ \text{reflected: } \mathbf{k}' &= \mathbf{k}^* \equiv -\mathbf{k} + 2(\mathbf{k}\mathbf{n}_X)\mathbf{n}_X, \mathbf{p}' = \mathbf{p}^* \equiv -\mathbf{p} + 2(\mathbf{p}\mathbf{n}_X)\mathbf{n}_X, \end{aligned} \quad (16)$$

with  $\mathbf{n}_X \equiv \mathbf{p}_X/|\mathbf{p}_X|$ . Since these two possibilities interfere, the double-twisted cross section will depend not only on  $|\mathcal{M}(\mathbf{k}, \mathbf{p})|^2$  but also on  $\mathcal{M}(\mathbf{k}, \mathbf{p})\mathcal{M}^*(\mathbf{k}', \mathbf{p}')$ , the *autocorrelation function* of the amplitude. Note that such a quantity is inaccessible with plane wave scattering.

Again, skipping the details which can be found in [5] we show the result for the cross section:

$$d\sigma_{2rw} = \frac{1}{8\pi \sin(\delta_k + \delta_p)} \int d\phi_k d\phi_p \frac{j_{PW}(\mathbf{k}, \mathbf{p})}{j_{2rw}} [d\sigma_{PW}(\mathbf{k}, \mathbf{p}) + d\sigma'_{PW}(\mathbf{k}, \mathbf{p})], \quad (17)$$

where  $d\sigma_{PW}(\mathbf{k}, \mathbf{p})$  is the usual plane wave cross section, while

$$\begin{aligned} d\sigma'_{PW}(\mathbf{k}, \mathbf{p}) &= \frac{(2\pi)^4 \delta(E_i - E_f) \delta(p_{zi} - p_{zf}) \delta^{(2)}(\mathbf{k} + \mathbf{p} - \mathbf{p}_X)}{4E_p \omega j_{PW}} \\ &\times \text{Re} \left[ e^{2im(\phi_k - \phi_X) + 2in(\phi_p - \phi_X)} \mathcal{M}(\mathbf{k}, \mathbf{p}) \mathcal{M}^*(\mathbf{k}', \mathbf{p}') \right] d\Gamma_X. \end{aligned} \quad (18)$$

and

$$\delta_k = \arccos \left( \frac{\mathbf{p}_X^2 + \kappa^2 - \eta^2}{2|\mathbf{p}_X|\kappa} \right), \quad \delta_p = \arccos \left( \frac{\mathbf{p}_X^2 - \kappa^2 + \eta^2}{2|\mathbf{p}_X|\eta} \right).$$

Note that, in contrast to the single-twisted case, the double-twisted cross section is  $m, n$ -dependent and, similarly to the single-twisted case, stays finite at small  $\kappa, \eta$ .

## 5 Conclusions

Orbital angular momentum (OAM) is a new degree of freedom, which can be used in high-energy physics to gain more insight into properties of particles and their interactions. In this contribution we discussed general aspects of high-energy collisions involving particles with non-zero OAM in the initial state.

The single-twisted (i.e. OAM state collision with a plane wave) cross section is represented via the plane wave cross section averaged over the azimuthal angle of one of the incoming particles. If the initial twisted particle is prepared in a superposition state with

different orbital quantum numbers, a Fourier analysis of the plane wave cross section with respect to the initial azimuthal angle can be performed. The expression we found for the double-twisted cross section involves not only the plane wave cross section, but also the autocorrelation function of the amplitude. It is interesting now to see what new insights into specific high-energy processes can be inferred from these possibilities.

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## References

1. S. Franke-Arnold, L. Allen, M. Padgett, “Advances in optical angular momentum”, *Laser and Photonics Reviews* **2**, 299 (2008);  
A. M. Yao and M. J. Padgett, “Orbital angular momentum: origins, behavior and applications”, *Advances in Optics and Photonics*, **3**, 161-204 (2011).
2. U. D. Jentschura and V. G. Serbo, “Generation of High-Energy Photons with Large Orbital Angular Momentum by Compton Backscattering”, *Phys. Rev. Lett.* **106**, 013001 (2011) [arXiv:1008.4788 [physics.acc-ph]].
3. U. D. Jentschura and V. G. Serbo, “Compton upconversion of twisted photons: backscattering of particles with non-planar wave functions”, *Eur. Phys. J.* **C71**, 1571 (2011). [arXiv:1101.1206 [physics.acc-ph]].
4. M. Uchida and A. Tonomura, “Generation of electron beams carrying orbital angular momentum”, *Nature* **464**, 737 (2010);  
J. Verbeeck, H. Tian, P. Schlattschneider, “Production and application of electron vortex beams”, *Nature* **467**, 301 (2010);  
B. J. McMoran et al, “Electron Vortex Beams with High Quanta of Orbital Angular Momentum”, *Science* **331**, 192 (2011).
5. I. P. Ivanov, “Colliding particles carrying nonzero orbital angular momentum”, *Phys. Rev.* **D 83**, 093001 (2011) [arXiv:1101.5575 [hep-ph]].
6. K. Y. Bliokh et al, “Angular momenta and spin-orbit interaction of nonparaxial light in free space”, *Phys. Rev.* **A82**, 063825 (2010).
7. M. R. Dennis, K. O’Holleran and M. J. Padgett, “Singular Optics: Optical Vortices and Polarization Singularities”, *Progress in Optics* **53**, 293-363 (2009).
8. S. D. Bass, “The spin structure of the proton”, *Rev. Mod. Phys.* **77**, 1257-1302 (2005);  
F. Myhrer, A. W. Thomas, “Understanding the proton’s spin structure”, *J. Phys. G* **G37**, 023101 (2010).
9. E. Leader, “On the controversy concerning the definition of quark and gluon angular momentum,” *Phys. Rev.* **D83**, 096012 (2011). [arXiv:1101.5956 [hep-ph]].
10. L. Allen, M. J. Padgett, M. Babiker, “The Orbital Angular Momentum of Light”, *Prog. Opt.* **39**, 291 (1999).